Smart Innovation, Systems and Technologies

Volume 91

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Algebraic Formalization of Smart Systems

Theory and Practice

Springer

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ISSN 2190-3018 ISSN 2190-3026 (electronic)
Smart Innovation, Systems and Technologies
ISBN 978-3-319-77050-5 ISBN 978-3-319-77051-2 (eBook)
https://doi.org/10.1007/978-3-319-77051-2

Library of Congress Control Number: 2018933497

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

In 1937 L. Bertalanffy proposed the concept of a System and the development of a mathematical apparatus for describing systems. In 1970s A.I. Mal'tsev developed a theory of algebraic systems connecting algebra and logic for studying algebraic and logical objects. In 1990s the concept of purities by predicates was introduced by one of the authors and we found out some applications of this concept to practice. This conception based on the theory of algebraic systems allows to deep and clarify connections between quantitative and qualitative analysis of a system.

The book which is offering to you, "The Algebraic Theory of Smart Systems. Theory and practice", is an attempt to reveal the general laws of the theory of Smart systems with the help of a very powerful and expressive language of algebraic formalization and also an effort to use this language to substantiate practical results in the field of Smart systems, which previously had only an empirical justification. In fact, this is a translation of the theory of Smart systems from verbal language to a much more expressive language of algebraic formalization allowing in a different light to see the laws of the theory of Smart systems is proposed to the reader.

The key users of this book are persons which using elements of artificial intelligence in their work.

Moscow, Russia Natalia Serdyukova
Moscow, Russia Vladimir Serdyukov

Acknowledgements

We are grateful to Professor Lakhmi C. Jain for his proposal to write this book, for his constant help and support, vision and contribution.

Thanks are due to the publisher for assistance during the preparation of the manuscript.

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Introduction

The emergence of ideas and Smart technologies has changed the mentality of human society and, in particular, in the field of human communication, i.e., in the sphere of universal and public relations. This change is connected with the appearance of a more expressive language—the language of digits and digital technologies of building connections. At present, Smart technologies and Smart systems have become a common phenomenon in almost all spheres of human life. In 1937, Ludwig von Bertalanffy proposed the concept of a system approach and a General Theory of Systems and also the development of a mathematical apparatus for describing typologically dissimilar systems. His main idea is to recognize isomorphism, that is identity, sameness of laws governing the functioning of system objects. In the 1970s, A. I. Mal'tsev developed a theory of algebraic systems that connects algebra and logic and which is a universal mathematical apparatus for studying both algebraic and logical objects. In 1990s the concept of purities by predicates was introduced by one of the authors, and later on we found out some applications of the theory of purities by predicates to practice. This conception makes possible to get a new methodology for the study of systems theory based on the idea of formalizing a notion of a system using algebraic systems and methods of general algebra. It allows to clarify connections between quantitative and qualitative analysis of a system in order to specify the previously known concepts in the deepening of the study of qualitative properties. The book which is offered to you, "The Algebraic Theory of Smart Systems. Theory and practice," is an attempt to reveal the general laws of the theory of Smart systems with the help of a very powerful and expressive language of algebraic formalization and also an effort to use this language to substantiate practical results in the field of Smart systems, which previously had only an empirical justification. In fact, this book is a translation of the theory of Smart systems from verbal language to a much more expressive language of algebraic formalization. It allows in a different light to see the laws of the theory of Smart systems.

It is well known that the management uses the following abbreviation SMART, which has been proposed by G. T. Doran in 1981. The abbreviation SMART means a smart target and combines capital letters from English words indicating what the real goal should be: Specific, Measurable, Attainable, Relevant, Time-bounded. Thus, SMART is a well-known and effective technology for setting and formulating goals. The English word smart has the following meanings when translating it into Russian: intellectual, intelligent, reasonable, elegant, clever, strong, sharp, and some other meanings. If we ignore the generally accepted interpretation of the word "smart" as an intellectual, or the word "SMART", originally used in the management, then we can say proceeding from the generally accepted axiomatics of Systems Theory, that in fact any system in its functioning has one of its goals its highest, or, more precisely, the optimal level of development, that is, the level of smart. Recently, various scientific schools, for example, Russian statistics school, began to consider SMART as a fundamentally new social process. In this regard, the title of proposed monograph is as follows: Algebraic Formalization of Smart Systems, Subtitle: Theory and Practice. The subtitle is explained by the original purpose of the theory of systems, like any abstract theory—its ability to be really in demand and used by the human society. This key position for us largely determined the title of the book and its content.

Now let us give a brief content of the book.

We begin our considerations in Chap. 1 with three basic questions:

- what is the meaning of the concept of "formalization"?
- how to build a formalization, allowing one to obtain and justify meaningful result in the General Theory of Systems, not limited to empirical reasons?
- how to interpret the results of general systems theory to specific spheres of human activity, such as the general theory of training, IT technology, economy, and finance, as the link with almost all types of modern human activities (e-learning, IT, economics, and finance)?

The key point for us to answer these questions is the connection between logic and algebra. In 1986, I. R. Shafarevich observed that the present period of development of sciences is characterized by the mathematization of the science. Algebra has always occupied a leading position in mathematics. So, in Chap. 1 we shall consider the following main points:

1.1 The Concept of Formalization as a Tool to Study the Phenomena, Processes, and Practical Outcomes on a Theoretical Level
1.2 Two Directions of the Development of Logic. From Deductive Systems to A. I. Mal'tsev's Systems
1.3 Algebraic Formalization of the General Concept of a System, Based on Factors Determining a System
1.4 The Hierarchy of Algebraic Formalization
1.5 Probabilistic Algebraic Formalization

1.6 A Series of Distribution of a Complete Countable Distributive Lattice of Algebraic Systems. A Distribution Function of a Random Function of a Lattice of Algebraic Formalizations
1.7 Examples of Usage of Hierarchies of Algebraic Formalizations

The development of system approach and a lot of works devoted to the results in general system's theory brought up the question of what language these results should be expressed and how these results should be justified. System approach specifies General scientific methodology, so the justification of the results in this area should not be only empirical. More and more works of different complexity and different expressive means that offer various formal languages and approaches to describe the general system theory appear.

Here we give a commonly accepted intuitive definition of an abstract system used in many of the already known attempts to formalize a concept of a system. The system is the minimum set of elements connected by a certain structure which gives this set of elements certain qualities that ensure the achievement of the system's goal. In Sects. 1.1–1.3, we consider, from the position of algebraization of logic, the history of the question of the uprising of various formalizations in mathematics and show the transition from deductive systems to Malt'sev's algebraic systems in order to explain our idea of formalization. In Sect. 1.4, the expressive properties of various formalizations are considered and then a hierarchy of algebraic formalizations is constructed. In points 1.5–1.7 we defined the lattice of logics $\boldsymbol{L} = \langle L, \cup, \cap \rangle$ as a lattice of algebraic systems, concepts of a random formula, the probability of a random formula, the distribution function of a random formula, a random function of a lattice of algebraic formalizations in order to have an opportunity to monitor changes in a system's functioning.

In Chap. 2 we consider the performance of a system S by using an algebraic system A_S of factors determining the System S, P-properties of a system, where P is a predicate defined on a class of algebraic systems closed under taking factor systems and subsystems. The main points of this chapter run as follows:

2.1 Factors Determining the System
2.2 The Scheme of the Dynamic Predicates' Functioning in Models That are Groups
2.3 Cycles in the System's Development and Functioning
2.4 Algorithm for Determining and Regulating Smart System's Properties

We introduce in this chapter a method for modeling the final states of the system and determining the number of final states using the technique of group theory and the notion of purities by predicates or P-purities. Predicate P, i.e., function with a set of values from two elements $\{0, 1\}$ or $\{\mathit{false}, \mathit{true}\}$, that is, in fact, a condition that determines some property of the set separates the static properties of the system if it does not depend on time or on changing other external in relation to the system of factors. For example, for a class of algebraic systems, or for a class of all groups, or for a class of all abelian groups, the property of purity is a static one. Predicate P can single out dynamic properties of the system if it depends upon time or upon the changing of other factors external to the system. For example, if we consider financial systems and knowledge systems about them, then predicates that highlight financial sustainability, the legal sector of the economy, etc., are dynamic ones; that is, they depend on time, on the changing internal conditions of functioning of society. For learning systems, predicates that highlight levels of learning complexity which, in turn, depend upon the degree of development of society, and so on, are dynamic. Predicates, in contrast to numerical indicators, allow us to characterize the studied properties in a single integrated complex of both numerical indicators and links that are defined with the help of these predicates, and in the dynamic, if they are dynamic predicates, and in static if they are static predicates. An important question when studying the properties of a system and the process of its functioning, and in particular when studying the properties of a smart system and its functioning, is the question of how to determine that a system or a smart system ceases to satisfy some property P or some complex of properties Π. To answer this question, we introduce the notion of a partial probability measure on the set of unary predicates defined on the class of groups and closed with respect to taking subgroups and factor groups. In the point 2.2 in order to describe the change in the properties of the system during its operation and the possible change or adjustment of its target, we use dynamic unary predicates defined on the class of all groups. To characterize the functioning of dynamic predicates in models which are groups, we define a partial probability measure on the class of all unary predicates defined on the class of groups and closed with respect to taking subgroups and factor groups and consider binomial distribution for the realization of a complex of properties $\mathcal{P} = \{P_i | i \in I\}$ of a system S in n trials with the probability of successful realization of a complex of properties $\mathcal{P} = \{P_i | i \in I\}$ of a system S, equal to p, and partial binomial distribution for the realization of a complex of properties $\mathcal{P} = \{P_i | i \in I\}$ of a system S in n trials with the probability of successful realization of a complex of properties $\mathcal{P} = \{P_i | i \in I\}$ of a system S, equal to p. After that in points 2.3 and 2.4 we examine cycles in the system's development and functioning and construct an algorithm for determining and regulating smart system's properties.

In Chap. 3 we consider the simulation of the smart system with the help of finite group of factors determining the system, P-properties of the system, Cayley tables, and their role in modeling associative closed system with feedback. This chapter consists of the following points:

3.1 P-Properties of the Smart System. Sustainability of Smart Systems
3.2 Example. Smart Systems Modeling by a Group of Four Elements
3.3 Relationship between Factors Determining a System and Elements of a System
3.4 Substitution of Functions of a System. System's Compensational Possibilities
3.5 Compensational Functions of a Quotient-Flexible Smart System
3.6 Sustainability upon the Smart System Functioning
3.7 Loss Detection Point of Sustainability of a System Algorithm that Uses Models of Groups of Factors Describing the System

One of the most important issues of the Smart System Theory solutions for which the theory of finite groups can be used is a question about the sustainability of a system. Under the sustainability of the system we shall mean the system's ability to save its current state upon the influence of external and internal influences. Sustainability is a primary quality of any system. In the absence of this quality a system cannot exist. Sustainability brings together the various properties: resistance to external factors, sustainability, reliability, etc. In this chapter, it is from this position we shall begin to consider the question of the sustainability of the system which is defined in fact by the internal structure of its connections, robust, and interchangeability of structural resources. Any algebraic relations in the group $\boldsymbol{G}_S$ which is a group of factors determining system S defines communications in the system S and so the system's sustainability to some extent. If, for example, we have a system of equations

$$\underset{i \in I}{\Lambda} w_i(x_1, \ldots, x_{n_i}) = e,$$

then we can consider that it represents some connections between the elements of set of its solutions in $\boldsymbol{G}_S$.

Exactly from this position to study the property of sustainability of system, the notions of quotient-rigid and quotient-flexible systems are introduced. In the Chap. 3 with the same position we propose the following partial classification of the property of the sustainability of the system, which complemented the concept of P-quasi-sustainable system:

- the compensational sustainability or the factors' sustainability of the system S for the interchangeable factors a_i and a_j for the quotient-flexible systems,
- the sustainability with regard to the system's target of the system S which is described by the finite group of factors $\boldsymbol{G}_S$,
- the quasi-sustainability with regard to predicate which includes as a special case the sustainability with regard to the system's target,
- the final sustainability of the system,
- the compensational sustainability or the factors' sustainability of the system S for the interchangeable factors a_i and a_j for the quotient-flexible systems,
- the sustainability with regard to the system's target of the system S which is described by the finite group of factors $\boldsymbol{G}_S$,
- the quasi-sustainability with regard to predicate which includes as a special case the sustainability with regard to the system's target,
- the final sustainability of the system which we shall consider in the Chap. 10.

In Chap. 3 we also show that we can restrict the study of the infinite system by the usage of finite sets, namely finite sets of factors that determine the system.

In the Chap. 10 we shall establish a connection between the concepts of the final sustainability of the system and Lyapunov sustainability of the system.

After that in point 3.2 we consider an important question that arises during the study of the properties of the sustainability of the system is the issue about the possibility of mutual substitution of elements of the system, or the factors determining the system or the system's functions to achieve system goals. We consider this question in the present chapter. In order to outline possible solutions of this issue, we start firstly from the question about the relationship between the factors which determined the system and elements of the system, and then, on this basis, the issue of compensational properties of the system is considered. During this consideration, we outline the following main items: 3.2–3.7. In item 3.3 we show how one can use only finite sets to study infinite systems.

In the examples, we have considered the representation of a system S by a group of factors $\boldsymbol{G_S}$ where group $\boldsymbol{G_S}$ was finite. During the study system's properties directly the following question arises: How one can link the factors which determine the system and system's elements?

To solve this problem we shall use the following procedure.

Let system S consist of the following elements: $S = \{s_\propto |\ \propto \in \Lambda\}$.

Suppose that the group's of factor determining the system S main set is $G_S = \{e, a_1, \ldots, a_n\}$, and herewith the elements of a system S, defines each factor that is mutually relevant to each factor are set off:

$a_i \leftrightarrow S_i = \{s_{\propto_i} |\ \propto_i \in \Lambda_i, i = 1, \ldots, n\} \neq \emptyset.$

Note that each subset $S_i = \{s_{\propto_i} |\ \propto_i \in \Lambda_i, i = 1, \ldots, n\}$ of the set S one-to-one corresponds the subset $\{\Lambda_i | i = 1, \ldots, n\}$ of the set Λ.

The following cases are possible:

(1) $\{\Lambda_i | i = 1, \ldots, n\}$ is a splitting of a set Λ. This means that $\bigcup_{i=\overline{1n}} \Lambda_i = \Lambda$, and $\Lambda_i \cap \Lambda_j = \emptyset$ for any $i, j \in \{1, \ldots, n\}$ such that $i \neq j$.

(2) $\{\Lambda_i | i = 1, \ldots, n\}$ is not a splitting of a set Λ. By virtue of the definition of the group of factors which determine the system one can assume without loss of generality that $\bigcup_{i=\overline{1n}} \Lambda_i = \Lambda$. So there exist $i, j \in \{1, \ldots, n\}$ in this case such that $\Lambda_i \cap \Lambda_j \neq \emptyset$. In this case, we construct the grinding of the set $\{S_i | \mathrm{i} = 1, \ldots, \mathrm{n}\}$ up to the splitting $\{S'_\mathrm{i} | \mathrm{i} = 1, \ldots, \mathrm{n}\}$ of the set S, where $S'_i = S_i \backslash \left(\bigcup_{j=1, j \neq i}^{n} (S_i \backslash S_j) \right), i = 1, \ldots, n.$

If the condition (2) takes place then the intersection of clusters $S_i \cap S_j$ is called a reserve of functions f_i and f_j.

Conditions (1) and (2) lead to the following definitions.

Definition The system S is called a quotient-rigid one if the condition (1) is true. The system S is called a quotient-flexible one by factors and a_j if $\Lambda_i \cap \Lambda_j \neq \emptyset$, $i, j \in \{1, \ldots, n\}$.

The assumption of the finiteness of the group of factors G_S which determine the system S is not essential.

Thus, without loss of generality we can assume that each factor $a_i, i \in I$ from the group of factors G_S which determine a system S corresponds to a cluster of elements $S_i = \{s_{\propto_i} |\ \propto_i \in \Lambda_i\}$ of a system S. This correspondence is one-to-one.

In item 3.4 under the substitution (compensation) of a broken function of a system, we would understand the adaptation of the system to changing conditions of its existence and a replacement as a consequence of this broken or ineffective or not working elements of a system by relatively more efficient elements of a system. We would call such elements of a system as follows: substitutional elements or compensational elements. After that in item 3.5 we prove the following theorem.

Theorem *If the smart system S is a final sustainable one, then the elementary theory* $Th(\boldsymbol{G_S})$, *where* $\boldsymbol{G_S}$ *is a group of factors that determine the system S, is a complete one.*

Then in item 3.6 the question about how to describe the following situation in the functioning of a system is examined. Suppose that some non-empty set of elements of a smart system are out of order in the process of functioning of a system but, however, a system continues to function through its other resources and achieves its purpose. We shall offer the description of this situation for the case when the smart system S is described by a finite groups of factors $\boldsymbol{G_S}$. The next question which is considered in item 3.7 is the following. When modeling any system, the question of how to determine possible points of crisis in their functioning arises. The main question which arises here is the following one. Let $\boldsymbol{G_S}$ be a model of algebraic formalization of a system S not detecting or in other words not noticing the onset of the crisis point in the development of a system. The question of how we should change or supplement the model $\boldsymbol{G_S}$ in order that it would be able to predict or to "see" the onset of a possible crisis arises. The theory of catastrophes and the theory of bifurcations give an answer to this question for continuous models.

We propose the algorithm to determine possible points of crisis in our case, the case of discrete models of algebraic formalization of smart systems to use.

In Chap. 4 we consider the basic properties that determine the system: integrity, internal, and external attributive features that determine the system, that is allocating this system from all others. Then the integrity property is generalized to the case of P-integrity and P-internal and P-external attributive characteristics of the system, allowing to classify the properties of the system according to their various components. The formalization of the system goal made it possible to introduce the notion of a quasi-stable system with respect to the property P and the innovation system with respect to the property P. The essence of the system approach runs as follows: All the elements of the system and all operations in it should be considered only as one whole, only as an aggregate, only in interrelation with each other. Moreover, in constructing the algebraic formalization of smart systems, we shall fully take into account the Gödel incompleteness theorem, the essence of which is that it is impossible to describe the system by using the means of this system only. Therefore, to formalize smart systems, we apply a factor approach, corresponding definition in Chap. 4. Besides it, in this chapter we introduce the notion of external attributive features of the system and internal attributive features of the system, with the help of which we shall formalize the axiomatic of smart systems. As well a hierarchy of different levels links of the system is constructed. A theorem on the description of the system's links is proved. In addition, on the base of the theory of binary relations constructed by A. I. Mal'tsev, a classification of the binary relations of a system of each finite level is upbuilded. The obtained results are applied to models describing the system's synergistic effects and the processes of system's decomposition and synthesis. Chapter 4 consists of the following items:

4.1 Introduction
4.2 System Approach Basic Principles. System's Links. Connection with Synergetics
4.3 The Model of Hierarchy of Structural Links of the System
4.4 Types of System Connections. Different Types of Classifications. Classification of Binary Links of the First Level of the System
4.5 Closed Associative Systems with Feedback Partial Classification on the System Links Levels and the Number of Synergistic Effects
4.6 System Binary Links and Mappings
4.7 Algorithm of Analysis and Decomposition of the System by its Links Levels
4.8 Example. System Decomposition. Smart System THE World University Rankings. Evaluation of THE World University Rankings system
4.9 Algebraic Formalization of the Axiomatic Description of Smart Systems

The formalization of the axiomatic of smart systems we begin in item 4.2 with a review of the basic principles of the systems approach, the study of which requires the use of a synergistic approach. These are the following principles:

– the aggregate of the system's elements is considered as one whole, possessing a set of definite links and properties. So it turns out that the system is not a simple union of its elements. It is necessary to take into account the links between the elements of the system, providing certain properties of the system, that is the structure of the system;
– the properties of the system are not simply the sum or the union of the properties of its elements. The system can have special properties, which may not exist for the individual elements that arise due to the connections between the elements of the system, that is, due to the structural links of the system. The researcher, using a system approach, first decomposes the system into subsystems and elements, determines the goals of their functioning, the criteria for evaluating their effectiveness, builds models for their functioning, and then sequentially synthesizes them into the system model. This problem is extremely complicated, because the structure of system's links is not known a priori to the researcher. The structure of the system's links is closely related to synergistic effects. In item 4.3 we shall first concentrate on the hierarchy of the system's links. The main theorem of this item runs as follows.

Theorem about the description of the system's links.
Links of the level no more than n of the system S, where n is a natural number, are determined by no more than two combinations of connections of the level no more than n of the system S.

In item 4.4 we consider operations over system links and introduce a concept of a group of all links of the level n of a system S. After that in item 4.5 we noticed that the construction of an exhaustive detailed classification of closed associative systems with a feedback even over the levels of the system's links is hardly possible at the present time. Therefore, we consider here a special case. A partial classification will be made on specific examples which show how one can act in the general case within the framework of the assumptions made. Let us make the following remark. It is possible to classify the finite models $\boldsymbol{G_S}$ of factors which determine the system S, in the case when for each positive integer n the links of the system of level n has a finite group structure, that is, $\boldsymbol{C_n(S)} = \langle C_n(S), \circ, \square^{-1}, e \rangle$ is a group, since a complete description of finite groups has now been obtained.

Definition The system S is called factor-fractal by levels i, j, if the group of links $G_i(S)$ of level i is isomorphic to the group of links $G_j(S)$ of level j of this system.

Such a fractality we encounter, for example, in biology when transferring properties from parents to offspring. In item 4.6 it is shown that an important role in the process of decomposition is played by the split-off the links levels of the system, since the separation of links levels in the system is in fact decomposition. In 4.7 on the base of previous item, an algorithm of analysis and decomposition of the system by its links levels has been constructed. In item 4.8 we begin the investigation system decomposition on the example of the smart system THE World University Rankings. Smart system THE uses 13 parameters (or, evaluation criteria) with weights, which are expressed in percentages from the total score on the several categories of evaluation criteria. There are five categories (or, blocks) in this ranking system; as a result of previous theorem we get that this ranking is a rather sustainable one:

Let us consider a system $\boldsymbol{S}$ which represents THE World University Rankings. A decomposition of this system gives five subsystems, namely S_1, S_2, S_3, S_4, S_5; they correspond to each of the five mentioned above categories. Let G_S be a group of factors, which represent the system S. Let B_1, B_2, B_3, B_4, B_5 be respectively groups of factors, which represent subsystems S_1, S_2, S_3, S_4, S_5. We may apply the additional restriction on system S and subsystems S_1, S_2, S_3, S_4, S_5—the operation of composition of the factors is a commutative one. Under this restriction, a synthesis of system S is described by the following theorem.

Theorem *Let the operation of composition of factors which represent the closed associative system with a feedback be a commutative one. Then the synthesis of the systems S_1, S_2 is described by the group of factors $Ext(B_2, B_1)$,*[1] *the synthesis of the systems S_1, S_2, S_3 is described by the group of factors $Ext(B_3, Ext(B_2, B_1))$, the synthesis of the systems S_1, S_2, S_3, S_4 is described by the group of factors $Ext(B_4, Ext(B_3, Ext(B_2, B_1)))$, the synthesis of the systems B_1, B_2, B_3, B_4, B_5 is described by the group of factors $Ext(B_5, Ext(B_4, Ext(B_3, Ext(B_2, B_1))))$.*

[1] The group of extensions of an abelian group B_1 by the abelian group B_2.

Because the numbers of factors (that represent a close associative system S with a feedback with commutative operation of composition of factors) is finite, then, in this case—to some extent, there are some theorems which allow to simplify the synthesis process of the system S. In 4.9 the axiomatic description of the system and its formalization is given. Formalization of the system's properties using the predicates is considered also.

In Chap. 5 we consider the following questions: different approaches to the definition of duality in Smart Systems Theory, measurement of the system's links strength, the group of links of a system as a group defined on the Cayley graph of the system, the concept of efficiency and its formalization, the concept of *P*-efficiency of a system, *P*-subgroups of effective links of a system. Chapter 5 consists of following items:

5.1 Preliminary Facts
5.2 Several Examples
5.3 System Connections Strength. Example: The Social Relationships Strength
5.4 Duality in System Theory
5.5 The Connection Between Duality and the Concept of a Factor of a System
5.6 Algebraic Formalization of Modeling the Processes Preserving the Operation of Composition of Factors of a Closed System
5.7 Duality in the Theory of Strong and Weak System's Links
5.8 Efficiency (Utility of a Smart System). Formalization of Efficiency
5.9 Presentation of the General Task of the Smart System Effectiveness Determining in the Form of an Optimization Problem with Risks
5.10 Examples. The Use of Duality for Complex Smart Systems Classification by the Number of System Goals. Stability by the Parameter of Achieving the Goal of the System

In addition to Chap. 4 in Chap. 5 we shall continue to study the links of the system and define another group of system's links as a group defined on the Cayley graph of the group of factors that determine the system. Then we proceed to study the subgroups of effective connections of the system. We shall also introduce the notion of a common efficiency problem with risks. The next question that we shall consider in Chap. 5 is the use of duality in systems theory. The question of duality, and, in particular, the question of duality in mathematics, is one of the most interesting questions connected, notably, with philosophy. One of the methods for studying duality in systems theory is the tensor method of dual networks. We shall give a brief survey of both of these methods. In item 5.2 we outline here the following examples and their brief survey:

1. Connections in social systems.
2. Connections in physical systems, string theory.
3. Connections in the tensor method of dual networks.

In item 5.3 the links strength indicators introduced in that explain Granovetter's theory. In item 5.4 we consider other versions of constructing a duality theory for the theory of systems proposed. An important role in this matter is played by the formalization of the concept of the connection of the system and the clarification of its meaning. Here several ways of formalizing the links of the system are proposed. The first way to construct a formalization of the system's links runs as follows. The visual representation of the connections of the system uses graph theory. We have constructed on this basis, a group of the system's links that uses the Cayley graph of the group of factors G_S, determining the system S and the construction of the free product.

The second way runs as follows. Let the system link connects some elements a, b of the system, and we are examining the model of factors which determine the system S. Let this model be an algebraic system $\boldsymbol{A_S} = \langle A_S, \Omega \rangle$ of the signature Ω. The system's links should preserve but not destroy the internal structure of the system. So, it is natural to consider the homomorphisms of the system $\boldsymbol{A_S}$ into itself, that is the maps of the set A_S into itself, preserving operations and predicates from Ω, as the system's links.

Hence from we obtain several ways to study duality in smart systems theory.

The first way of constructing duality for the theory of smart systems uses models of factors that determine the system, and these models of factors are algebraic systems $\boldsymbol{A_S} = \langle A_S, \Omega \rangle$ of some signature Ω. Further the classical theory of duality from category theory is used in this method, [1]. It follows from the existence for each category the dual one that there works a duality principle in the category theory, that is, for every true sentence of the predicate calculus with respect to one category there exists a dual true statement for the dual category. The statement Pr^D, which is dual to the statement Pr and is formulated in the language of category theory, is obtained by interpreting in the category $\Re$ the sentence Pr, considered in the dual category $\Re^D$. A dual statement is obtained from the original one by preserving the logical structure of the statement and replacing in its formulation all the arrows by the opposite, and all products of morphisms into products of morphisms written in the reverse order.

The second method was proposed by us for the case when the model of factors is a group of factors G_S. Here we can consider the following two cases.

The first case: The group of factors which determined the system S is finite, and $|G_S| = n$. It is well known that in this case the group $\boldsymbol{G_S}$ can be embedded in the symmetric group of all permutations $\boldsymbol{S_n}$ of degree n. The second case does not use restrictions on the number of elements of the group $\boldsymbol{G_S}$. In the second method, we propose to embed $\boldsymbol{G_S}$ in its holomorph $Hol\boldsymbol{G_S}$. First of all let us consider the case where $\boldsymbol{G_S}$ is a finite abelian group. Then the holomorph $Hol\boldsymbol{G_S}$ of the group $\boldsymbol{G_S}$ is a semidirect extension of the group $\boldsymbol{G_S}$ with the help of its group of automorphisms $Aut(\boldsymbol{G_S})$. Let us use the multiplication form of the record for a group operation in $Aut(\boldsymbol{G_S})$, and for a group operation in $\boldsymbol{G_S}$ and in $Hol\boldsymbol{G_S}$ let us use $\circ$ and +, respectively. The main set of the group $Hol\boldsymbol{G_S}$ can be considered as the set of all ordered pairs (g, φ), where $g \in \boldsymbol{G_S}$, $\varphi \in Aut(\boldsymbol{G_S})$. The group operation is given in $Hol\boldsymbol{G_S}$ by the rule: $(g, \varphi) + (h, \psi) = (g \circ \varphi h, \varphi\psi)$ for any $(g, \varphi) \in Hol\boldsymbol{G_S}$, $(h, \psi) \in Hol\boldsymbol{G_S}$.

In general holomorph of a group is the concept of group theory that aroses in connection with the solution of the following problem: Is it possible to include any given group $\boldsymbol{G}$ as a normal subgroup in some other group so that all automorphisms of $\boldsymbol{G}$ are consequences of inner automorphisms of this larger group? To solve this problem, we construct a new group $Hol(\boldsymbol{G})$, with respect to the group $\boldsymbol{G}$ and its automorphism group $Aut(\boldsymbol{G})$, whose elements are the pairs (g, φ), where $g \in \boldsymbol{G}$, $\varphi \in Aut(\boldsymbol{G})$, and in which the composition is defined according to the following formula:

$$(g_1, \varphi_1)(g_2, \varphi_2) = \left(g_1 \circ \varphi_1^{-1}(g_2), \varphi_1\varphi_2\right)$$

Herewith, the automorphisms $Aut(\boldsymbol{G_S})$ of the group of factors of the system $\boldsymbol{S}$ are in fact the links of the system $\boldsymbol{S}$ with special properties:

(1) the one-to-one correspondence between the factors that determine the system,
(2) the preservation by the link of the composition operation of the factors which determine the system.

In this connection, a special role here belongs to perfect groups, that is such groups $\boldsymbol{G}$ which are isomorphic to the group of its automorphisms $Aut(\boldsymbol{G})$. For example, $\boldsymbol{G} \cong \boldsymbol{S_n}$, where $n \neq 2, 6$. We have $Hol\mathbf{G}/\boldsymbol{G} \cong AutG \cong G$ for a perfect group $\boldsymbol{G}$. We obtain the following conclusion from all the above.

The main conclusion about duality.

The main idea of the representation of duality in the theory of systems runs as follows. Let $\{S_\propto |\propto\in \Lambda\}$ be a non-empty set of systems; and the element $a \in S_\propto, \propto\in \Lambda \underset{\square}{\Leftrightarrow} \propto\in \bigcap_{\propto\in\Lambda} S_\propto \neq \emptyset$. All such elements a give us in point of fact the set of all connections between systems $S_\propto, \propto\in \Lambda$. Elements of a new system S^d which is dual to the system S are the systems $S_\propto, \propto\in \Lambda$, that is the elements of the set $\{S_\propto |\propto\in \Lambda\}$.

In item 5.5 we return to Example 3 from Sect. 5.2, and namely to the definition of the tensor and the conjugate or dual vector space. According to the definition of the conjugate or dual vector space, we have $V' = \{f | f : V \to R\}$, where every f is a linear function from the vector space V into the field of real number R. Let us consider the correspondence

$$V' = \{f | f : V \to R\} \mapsto \{Imf \cong V/kerf \leq R\}$$

This correspondence, as well as the use of the concept of the group holomorph, helps us to introduce the concept of a factor system as a concept dual to the concept of a subsystem. This can be done for the model of algebraic formalization of the system by using the concept of factors which determine the system, and by using the concept of elements of the system for the system itself. In item 5.6 we consider several examples to clear up the process of decomposition of the education system by goals and links and after that the processes preserving the operation of composition of factors of a closed system and their formalizations. In the next item 5.6 we prove the following main theorem.

Theorem *Let S be a system and* $\boldsymbol{G_S} \cong \boldsymbol{V}$*, where* $\boldsymbol{V}$ *is an additive group of the Euclidean vector space of the dimension n, be a group of factors which determine the system S. Then the powers of links of the system S and the system* S' *dual to S and defined by the group of factors* $\boldsymbol{V}'$*, where* $\boldsymbol{V}'$ *is an additive group of the vector space* V' *which is conjugate or dual to the vector space V, are the same.*

In item 5.7 we introduce the definition of an efficiency function as an utility function. After that several examples of setting up the smart system efficiency function are considered. The next question that arises in the study of the utility function or the effectiveness of the system that we consider in item 5.8 is the question of how to link the effectiveness function and the effectiveness criterion with performance indicators. In item 5.9 we have noticed that the main conclusion about the representation of duality in the theory of systems allows us to tie up the proposed constructions to the classical approach of describing the properties of the system, which basically uses the notion of an element of the system, and not the factor which determines the system. This approach allows, for example, to classify complex smart systems S according to the number of goals of system S to study the stability of smart systems in terms of the parameter of achieving the goals of the functioning of the system.

In Chap. 6 we consider the following questions. We shall continue to study the concept of efficiency, and in line with this concept we introduce the definition of an innovative smart system, and in terms of algebraic formalization we shall describe the structure of innovative smart systems. To study the structure of the innovative system, the algebraic formalization of this concept will be used. In addition, we shall trace the analogy between the concept of an innovative smart system and the concept of an inverse limit, as well as analogues of the theorem on the description of abelian algebraically compact groups. Then we shall consider the concept of pseudo-innovative system, dual to the concept of an innovative smart system, and we shall get a description of pseudo-innovative systems using algebraic formalization. The notion of a quasi-sustainable system is introduced by the analogy with the concept of quasi-isomorphism from the abelian groups theory. An algebraic formalization of some properties of innovative smart systems and pseudo-innovative systems is constructed. Some examples of the use of these concepts consumed in the expert systems in training and in the economy are given.

After that we shall continue the empirical study of the process of system's decomposition using the example of the decomposition of the education system on the basis of these questions. The algorithm for a comprehensive assessment of the effectiveness of the functioning of the innovation system based on the tensor estimation of system performance is also proposed in Chap. 6. It is proposed to use homomorphisms of the group $\boldsymbol{G_S}$ of factors defining the system S into the group $GL(n, \Delta)$ of linear homogeneous transformations of the vector space R^n as tensor estimates of the efficiency of the functioning of the system S. One can also consider homomorphisms of the group of factors $\boldsymbol{G_S}$ that define a system$S$ in the group $GL(n, \Delta)$ of linear homogeneous transformations of the vector space Δ^n over an arbitrary field Δ.

The following results were obtained as applications:

– economic systems. In an economics with the presence of the shadow sector, a system with full implementation of P-connections cannot work autonomously if P does not implement any connections of the shadow sector.
– expert systems in learning. Testing with the full implementation of the links and the oral examination with the full implementation of the links give the same result. The levels of the impact of the knowledge system on the student have been singled out and tabulated. A more detailed consideration is given to one of the fragments of the knowledge system's decomposition, and it makes possible to determine the exposure levels listed in the table. Expert systems, an algorithm for compiling a database of errors, an algorithm for compiling a knowledge base, a theorem on describing errors, and a theorem on describing correct solutions have been used for this purpose. Together with the works, which deal with the issues of test quality and the practice of test assessment of knowledge in the Russian Federation, this makes it possible to determine the levels of knowledge of students with a sufficiently high degree of reliability.

Chapter 6 consists of the following items:

6.1 Formalization of Innovation and Effectiveness Concepts
6.2 Algorithm for a Comprehensive Assessment of the Effectiveness of a Smart System
6.3 Example. Decomposition of the Education System. Approaches to the Study of the Effectiveness of the Education System
6.4 Decomposition of the Knowledge System. The Representation of the System of Knowledge in the Form of an Algebraic System
6.5 Decomposition of the System. Analysis and Synthesis of the Knowledge Base

In item 6.1, we consider basic properties of innovative systems and examples of P-pseudo-innovative system. Item 6.2 is devoted to the task of constructing a numerical estimate of the effectiveness of the functioning of the system. This task is extremely difficult from a mathematical point of view since its solution involves a quantitative assessment of the appearance of qualitative changes.

We shall construct a tensor estimate of the effectiveness of the functioning of the system as a homomorphism of a group of factors G_S, determining the system S into a group $GL(n, R)$ of linear homogeneous transformations of the vector space R^n.

After that we construct an algorithm of a complex estimation of efficiency of functioning of the innovation system and the model of innovation management.

Then we consider the following questions: quasi-sustainability of pseudo-innovative systems, examples and application to expert systems in e-learning, economic systems.

In item 6.3, we decompose the education system into the following components: knowledge system S_1 (information subsystem), methodological and methodical complex S_2 of the knowledge system S (an adaptive subsystem), the system of students S_3 (target subsystem, target audience) and examine one of the fragments of the decomposition of the knowledge system in more details. In item 6.4 we consider the question of the representation of a system of knowledge. The brief survey of formal models of the representation of a system of knowledge and non-formal models of the representation of a system of knowledge (semantic, relational) is given. And item 6.5 is devoted to the question of decomposition of the system and to the question of analysis and synthesis of the knowledge base that arises in practice. In this item we construct the following algorithms: algorithm of errors description, algorithm for compiling a knowledge base, and search and analysis of correct task solving algorithms. After that we consider analysis of pupils solutions and transition to the record of the solution in the language of the narrow predicate calculus language (NPC) from the recording of the solution in the group theory language.

In Chap. 7 we have noticed that the accumulation of new properties of the system is associated with bifurcations or with the appearance of a qualitatively different behavior of the system element when a quantitative change in its parameters takes place, in accordance with works by I. Prigozhin and I. Stengers. It is assumed that the probability of reliable prediction of new properties of the system is small at the time of bifurcation, where bifurcation is a kind of a system regeneration. Contradictions arise naturally in the process of system's development, and they are the reason for the perfection development of systems. From the Theory of Systems, it is known that it is impossible to speed up the development of the system by artificially introducing contradictions into it, since it is impossible to determine whether the system, as a result of their resolution, will bear the new qualities. In this regard, one of the most important questions in the Systems Theory is the question of risks description. In this chapter, an approach to the classification of system risks from the position of algebraic formalization of the system is considered. This approach made it possible to distinguish between regulated (internal) and unregulated (external) risks of a system. As it is known, the question about the existence of infinite systems is debatable one. However, in this framework it is shown that the set of unregulated risks of any infinite system has a power of continuum accurate up to the regulated risks. An algorithm for managing the internal regulated risks of the system is constructed for a system represented by a finite group of factors. The risk function r of the system is defined as a function dual to the probability measure in the framework of algebraic systems formalization. This allowed us to consider probabilistic spaces with risk. Chapter 7 begins with an analysis of known existing approaches to risks description. In this connection, the main attention is paid to the quantitative definition of risk, which follows from the Kolmogorov–Chapman equation. Let us remind that the Kolmogorov–Chapman equation describes operations which occur according to the scheme of Markov random processes. Some relationships between the Kolmogorov risk function $h(x)$ and the risk function $r(x)$ introduced in Chap. 7 are found. Examples of the distribution functions $F(x)$ for which the risk function $h(x)$ is multiplicative one are considered. The risks of changes in formalizations ("failure of formalization") of the system using the

Kolmogorov–Chapman equation for exponential distribution are calculated. The statistical definition of risk is considered. Chapter 7 also presents a model of linear programming with risk. A linear programming model with a risk can be used in practice. Examples of the use of algebraic formalization for describing systemic risk in the particular case when the system of factors determining the risk of a closed associative system is considered in conclusion. In the case when the factors determining the risk of the system form a complete group of events that are independent in aggregate at any time and have the same probability density satisfying a certain condition, a measure of systemic risk is proposed. Chapter 7 consists of the following items:

7.1 Known Approaches to the Mathematical Determination of Risk. The Kolmogorov Risk Function
7.2 The Presentation of the General Model of Multi-criteria Optimization Problem in the Form of Linear Programming Task with Risks
7.3 System Approach to Risk
7.4 Mathematical Model of Risk
7.5 The Use of the Theory of Infinite Products to Quantify Risks
7.6 The Connection Between the Kolmogorov Risk Function $h(x)$ and the Risk Function r
7.7 Regulated Risks. Semigroup of Systemic Risks. Description of the System's Risk Semigroup
7.8 Algebraic Approach to the Description of Risks. Internal and External Systems Risks. Systemic Risk or System Risk

In item 7.1 we give a brief survey of well-known approaches to the quantitative definition of risk. In item 7.2 we present the general model of multi-criteria optimization problem in the form of a linear programming taking the risks into an account. In item 7.3 in order to ensure generality, we use an axiomatic approach to define the function of risk. In item 7.4 we construct risk function as a function dual to probability measure and examine its properties. In item 7.5 we consider some properties of infinite products that help one to quantify risks. In item 7.6 we consider the simplest examples of probability distributions with a multiplicative risk function. In item 7.7 we consider an external risk of the system, risks of formalization changes for the exponential distribution. In item 7.8 we introduce the notions of internal and external systems risks, consider the examples of risks functions on a finite σ-algebras, and construct an algorithm for regulating the internal risks of the system. After that we consider some properties of risk, prove the theorem about the description of systemic risk, and explore some examples.

In Chap. 8 we consider the question of how from an infinite model of factors that determined the system S one can go to the finite model of factors $\boldsymbol{G_S}$ which determine the system S. A list of necessary information from the finite groups theory, useful in the study of certain features of the functioning of the smart system, is given in addition. Table of classification of system's properties by models of finite groups of factors that determine the system in which some system's properties are classified is constructed on this basis by the models of finite groups of factors determining the system. The question about risk modeling in a smart university also is considered in this chapter. The model of an algebraic formalization of six factors of the risks of changes in long-term period of a development of the smart system is constructed on the example of the smart university. The algorithm of search of points of regulation of the closed associative system's functioning on the example of the model consisting of six factors is shown in this chapter too. Chapter 8 consists of the following items:

8.1 The Transition from an Infinite Model of Factors that Determine the System to a Finite Model of the System
8.2 The Necessary Information from the Finite Groups Theory Useful in the Study of Some Features of the System's Functioning
8.3 The Model of an Algebraic Formalization of Risks of Changing the Scenarios of the Long-Term Development of a Smart System of Six Factors on the Example of a Smart University
8.4 A Selection of Factors to Determine Long-Term Risks of a System
8.5 Conclusions. Future Steps

In item 8.1 the construction which is proposed helps one to create the finite model G_S of the system S in the form of a finite group of factors determined the system S. In fact to construct a Cayley table for the group G_S one can act in two following ways:

(1) To use combinatorial methods. One should search the defining relations of the model G_S with the help of simple enumeration.
(2) To make a qualitative analysis of the factors which determined the system and on this basis to explain the relationships between them.

In item 8.2 we give some known and interesting facts from the theory of finite groups that can be useful in studying such properties of the system as the presence of synergistic effects, the number of possible variants of forecasts for the development of the system, the stability properties of the system. The main idea of item 8.3 which unites the further presentation of Chap. 8 is to show that, with the correct and timely regulation the process of the system's functioning, it becomes a smart system in the sense of the optimal system on the selected smart criteria, or, in other words, a smart optimal system. In the proposed model of algebraic formalization of risks of changing the scenarios for the long-term development of a smart system of six factors, the risk of formalization's change from the symmetric scenario to the cyclic scenario, and the tensor index of the effectiveness of the system performance on specific indicators, can be calculated by the algorithm proposed in this section, for example, for a smart university system. In item 8.5 some future steps are proposed to develop a methodology of SmU modeling as a system based on an algebraic formalization of general systems' theory, theory of algebraic systems, theory of groups, and generalizations of purities, and identify formal mathematical conditions for a system—in this case SmU—to become efficient and/or innovative.

In Chap. 9 we return again to the special case in which the factors affecting the system determine the group. In this case, the system is a closed associative system with feedback. This chapter consists of the following items:

9.1 Particular Case: Factors Affecting a System Determine a Group
9.2 The Group of Automorphisms of the Group of Factors that Determine the System
9.3 Direct and Inverse Spectra of Groups and their Limits
9.4 The Role of Profinite Groups in Algebra and Topology
9.5 Predicates Defined by Systems of Equations on the Class of Groups
9.6 Interpretation of Systems of Equations over Groups of Factors that Describe a Smart System
9.7 *P*-topology
9.8 Pro-*P*-algebraic Systems

In item 9.1 we consider the meaning of the *P*-pure embeddings and several examples of *P*-purities in the class of all groups. In item 9.2 here we dwell briefly upon the modeling of "identical" factors with respect to the structure that act on the system. The question arises as to how all possible structures of connections between factors acting on the system can be described. We shall use the automorphism group of the group of factors that determine the system to this purpose.

After that we recall the definition and basic information about algebraically compact groups that are necessary for the study of innovative and pseudo-innovative systems. Algebraically compact groups are in some way a generalization of divisible groups in two following directions: The first line (1) is distinguished as a direct summand from the group containing it, when (2) certain conditions are imposed on how the subgroup is contained in the overgroup. If a divisible group can be defined as a group distinguished as a direct summand from any group that contains it, then an algebraically compact group is a group distinguished as a direct summand from any group that contains it as a pure subgroup. In item 9.3 we present some needed facts about direct and inverse spectra and their limits. In item 9.4 we recall the definition and basic information about profinite groups necessary for studying the formalization of innovative and pseudo-innovative systems. A topological group that can be represented as a projective limit of finite groups is said to be a profinite one. The class of profinite groups coincides with the class of compact completely disconnected groups. The concept of the profinite group has been time and again generalized; see, for example, Colin David Reid's paper about finiteness properties of profinite groups. Thus, for example, classes of pro-p-groups, where p is a prime number, pro-π-groups, where π is the set of prime numbers, pronilpotent groups, and pro-solvable groups were defined. In this item we introduce the notions of *P*-finite groups and pro-*P*-groups and construct some aspects of the analogous theory to the theory of algebraically closed abelian groups. The main theorem runs as follows.

Theorem *Let P be a predicate given on a class of groups which is closed under taking subgroups and factor-groups. Every group G, which is satisfied to predicate P can be embedded into pro-P-completion* $\widehat{G_P}$ *of a group G.*

In item 9.5 we change the definition of the purities on predicates, eliminating the condition that the predicate P should be closed with respect to taking the subalgebras. The main definition runs as follows.

Definition A subalgebra $\bar{B} = \langle B | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ of an algebra $\bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ is called a P-pure subalgebra of an algebra $\bar{A}$, if every homomorphism $\bar{B} \xrightarrow{\alpha} \bar{C}$ from a subalgebra $\bar{B}$ of an algebra $\bar{A}$ into an algebra $\bar{C}$ of a signature $\{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \}$, such that $P(\bar{C})$ is true, where predicate P is sustainable with respect to factor-algebras, can be continued up to homomorphism from algebra $\bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ into an algebra $\bar{C} = \langle C | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$, that is the following diagram is commutative:

$$\begin{tikzcd}
0 \arrow[r] & \bar{B} = \langle B | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle \arrow[r, "\varphi"] \arrow[d, "\alpha"'] & \bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle \arrow[dl, dashed, "\beta"] \\
 & \bar{C} = \langle C | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle &
\end{tikzcd}$$

It means that $\beta\varphi = \alpha$, where φ is an embedding $\bar{B} = \langle B | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ into $\bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$, P is a predicate defined on the class of algebras of the signature $\{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \}$, highlights the class of subalgebras which is closed under taking factor algebras[2] φ is called a P-pure embedding. For such a definition of the P-purity there will be no duality, which is analogous to the duality described by L. Fucks. In item 9.6 we consider examples of the description of the functioning of systems with the help of systems of equations over groups of factors. In item 9.7 we consider the well-known concepts of topology following P. M. Cohn and construct some analogues of definitions and theorems formulated and proved by him. And finally in item 9.8 we examine the definition of direct and inverse spectra of algebraic systems and their limits for the sequel and generalize some results of previous section to the common case of algebraic systems.

In Chap. 10 we marked that the question of the reliability of the obtained results is of great value for any theory. This is especially important when it comes to risk-free application of the theoretical results in practice. The reliability is especially significant for the humanities relating to the development and functioning of human society, such as pedagogy, the general theory of education, e-learning, economics, finance as their distinctive features are the following:

- impossibility of repetition of the experiment and frequently to perform the only experiment with sufficient accuracy, since there is always the human factor,
- the difficulty of collecting reliable and comparable statistical data in connection with the lack of standardized procedures.

[2]The main operations of the same type of algebraic systems of the same signature will be denoted in each of the algebras in the same way.

In this chapter we continue to study smart systems and, in particular, the concept of smart university in the context of theoretical justification of the results based on the algebraic formalization of the smart systems. The practical result of this investigation is the evaluation of sustainability of ranking universities systems.

Chapter 10 consists of the following items:

10.1 Sustainability: Ranking Systems
10.2 Final Sustainability of a System
10.3 Time Structure of Algebraic Formalization
10.4 The Algorithm of Determination of the Scenarios of Development of the System S and Points and Intervals of Loss of the Sustainability of the System S
10.5 The Connection between Notions of Final Sustainability, Stationary Points, and Classical Sustainability
10.6 Practice Example. Algebraic Formalization as a Tool of Assertion the Sustainability of Ranking Systems of an Evaluation of Activities of Universities

In item 10.1 we marked that in the study of system's functioning across the time and its ability to forecast changes of system's properties the question about system sustainability is rather important. This question is especially important for the Smart System Theory. The concept of sustainability is well studied in terms of the availability of various quantitative parameters describing the dynamic behavior of the system. There were introduced such concepts as Lyapunov sustainability, Zhukovsky sustainability. We shall consider discrete systems as in previous chapters. Under the sustainability of a discrete system, we shall understand its ability to return to the equilibrium position after the end of the action of external factors as in the case of continuous-time systems. To date the classification of such concepts as an equilibrium, as a notion of stationary point there were introduced. The indices characterizing the quality of discrete systems designed to evaluate the dynamic properties of the system, manifested in transient conditions, and to determine the accuracy of the system which is characterized by errors in the steady state after the transition was introduced. Dynamic indicators of quality characterize the behavior of free components of the transition process closed control systems or processes of an autonomous system. However, convenient integrated indicators which are a synthesis of qualitative and quantitative indicators of the phenomenon under study as such are absent. We propose to use Cayley table of a group G_S of factors determining the system S to characterize the quality of dynamics of the closed associative smart system with feedback S. This makes it possible to regulate the behavior of the smart system S in some cases. In item 10.2 we concern the notion of a final sustainability of a system. The link between the final sustainability and Lyapunov sustainability is reviewed. In item 10.3 the time factor is introduced into the construction of the group of factors G_S determined the system S to have an opportunity to characterize the scenarios of development of the system S. In item 10.4 an algorithm to determine the points (intervals) of the loss of a sustainability of a system S and scenarios of functioning of a system S is constructed. Examples of a usage of parametric statistic in part of laws of distribution of discrete random variables in an annex to the scenarios of development of the system S are discussed. An algorithm to define and regulate scenarios of system's functioning where a system is defined by a group of factors G_S of order p^2 for a prime number p is built. In item 10.5 the connection between of the notion of final stability, stationary points, and the classical notion of sustainability is discussed.

The main result of this section runs as follows.

Theorem *If a system is final sustainable, then it is Lyupunov sustainable.*

In item 10.6 as an example, we consider a way of formalizing a synthesis of a system by its decomposition with the usage the technique of the theory of extensions of abelian groups. After that on this basis we examine the sustainability[3] of the ranking systems of evaluation the effectiveness of universities.

This construction, that is ranking system, can be used for building ranking systems monitoring smart universities. Herewith, the blocks of ranking systems themselves will change, because in this case one will have to evaluate and compare: systems for monitoring the results of the educational process, expert communities, active educational technologies, modules of educational resources, quality of IT technologies, system of formation of individual educational trajectories, technology to determine the personality characteristics of a student, the effectiveness of financial support for the activities of a smart university, and others. This will help to create a monitoring of the education system that tracks the quality of education better than existing ranking systems of an evaluation of activities of universities. Using both of these theorems and the tensor estimate of system's functioning considered in Chap. 6, we can construct new ranking system to monitor and to manage Smart Education System. It is also important that it will help to make Smart Education System more sustainable.

Reference

1. Bucur, I., Delianu, A.: Theory of categories. In: General Algebra, vol. 2, p. 188. (1968) [Chapter 7: Categories]

[3]Let us explain the notion of a sustainability of a system once more. The system is a sustainable one if at withdrawing it from the external effects from the state of equilibrium (rest) it returns to it after the cessation of external influences. From the point of view of an algebraic formalization, it means that there are restrictions on the number of final states of the system.